Let

$$f(a,b,c) = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d(ijk) \tag{1}$$

and

$$g(a,b,c) = \sum_{gcd(i,j)=gcd(j,k)=gcd(k,i)=1} [\frac{a}{i}][\frac{b}{j}][\frac{c}{k}] \tag{2}$$

It is sufficient to prove that:
$f(a,b,c) - f(a-1,b,c) - f(a,b-1,c) - f(a,b,c-1) + f(a-1,b-1,c) + f(a-1,b,c-1) + f(a,b-1,c-1) - f(a-1,b-1,c-1)$
$= g(a,b,c) - g(a-1,b,c) - g(a,b-1,c) - g(a,b,c-1) + g(a-1,b-1,c) + g(a-1,b,c-1) + g(a,b-1,c-1) - g(a-1,b-1,c-1)$

Simplify the LHS:
LHS $= d(abc)$

Also simplify the RHS:
RHS
$= \sum_{gcd(i,j)=gcd(j,k)=gcd(k,i)=1}([\frac{a}{i}] - [\frac{a-1}{i}])([\frac{b}{j}] - [\frac{b-1}{j}])([\frac{c}{k}] - [\frac{c-1}{k}])$
= {the number of triplets of (i,j,k) such that gcd(i,j) = gcd(j,k) = gcd(k,i) = 1 and a%i = b%j = c%k = 0}

Fix a prime p. Let x be the maximal integer such that p^x divides a. Define y and z similarly.
The number of ways to decide p-factors of a divisor of abc is $x+y+z+1$ (any integer between 0 and $x+y+z$).
The number of ways to decide p-factors of i,j,k in RHS is $x+y+z+1$ ((0,0,0) or (positive,0,0) (x ways) or (0,positive,0) (y ways) or (0,0,positive) (z ways)).